Et

4AI3-31900 FR1

VOL. I

[redacted]

(NASA-CR-150393) NASTRAN HYDROELASTIC MODAL STUDIES. VOLUME 1: INTRODUCTION, THEORY, AND RESULTS Final Report (Universal Analytics, Inc.) 43 p HC A03/MF A01 N79-25351

CSCL 20D G3/34 Unclas 26861

1979
RECEIVED
NASA STI FACILITY
ACCESS DEPT.

UNIVERSAL ANALYTICS, INC.

Los Angeles

RECEIVED
NASA STI FACILITY

FINAL REPORT

NASTRAN HYDROELASTIC MODAL STUDIES

VOLUME I

INTRODUCTION, THEORY, AND RESULTS

Prepared for

National Aeronautics and Space Administration
Marshall Space Flight Center

Under Contract No. NAS8-31900

May 31, 1977

Prepared by

UNIVERSAL ANALYTICS, INC.
7740 West Manchester Boulevard
Playa Del Rey, California 90291

(213) 822-4422

FOREWORD

This document presents the Final Report on the development and use of a computer program for "Hydroelastic Modal Studies" for George C. Marshall Space Flight Center (MSFC) of the National Aeronautics and Space Administration.

This capability was developed as program additions to the NASTRAN (NASA Structure Analysis) computer program and delivered to MSFC for use in the analysis of the Shuttle ET tank hydroelastic modes.

Messrs. D. N. Herting, R. L. Hoesly, and D. L. Herendeen of Universal Analytics, Inc. were the primary contributors to the project. Mr. R. L. McComas of MSFC monitored the project and also contributed to the testing of the system. Messrs. D. Harper and J. Moreman of Computer Science Corp. performed valuable aid in implementing the system at MSFC.

The Final Report is divided into three volumes. This, the first volume, describes the theoretical basis of the system and results of the test cases. Also included are a brief summary of the project history and explanations and conclusions based on the results obtained in the contract. The second volume contains the Program Documentation and the third volume serves as a guidebook to the use of the system.

NASTRAN HYDROELASTIC MODAL STUDIES

TABLE OF CONTENTS

(Note: Each volume contains a detailed Table of Contents.)

VOLUME I - TABLE OF CONTENTS

VOLUME I - LIST OF FIGURES AND TABLES

Figures — Page

Tables

1.0 INTRODUCTION

This report describes the technical effort performed in the implementation of a general three-dimensional hydroelastic capability in the NASTRAN (NASA Structural Analysis) computer program. Although NASTRAN had provided capabilities for the analysis of compressible fluids with axisymmetric geometry, the Space Shuttle External Tank Analysis required more general capability and more efficient solution procedures. The basic approach described in this report extends the capabilities to provide for arbitrary fluid shapes, including tilted free surfaces, and allow for more efficient methods of obtaining the solutions.

One goal of the program development was to provide a general method for analyzing the combined mode shapes of arbitrary fluid and structure finite element models. The fluid is modeled with three-dimensional solid elements with options for tetrahedron, wedge, and hexahedron shapes. The elements are connected to fluid grid points which define the pressure in the fluid at the specified location. The structure may be modeled arbitrarily using the existing NASTRAN elements. The fluid/structure interface and the free surface are defined by the user with special NASTRAN boundary elements. A special purpose mesh generator program has been provided to generate the actual NASTRAN data cards for the fluid, the structure, and the boundary elements for typical tank-type models.

A second goal of the project was to formulate the matrices to provide efficient solutions for large-order problems. The structure matrices are processed separately and may be reduced using matrix condensation procedures (ØMIT) or through a modal formulation using the normal modes of the empty structure as solution coordinates. The fluid matrices are then transformed and connected to the reduced structure coordinates resulting in small, symmetric, solution matrices. This approach is particularly valuable when several different fluid levels are to be analyzed for one structure. The structure formulation and reduction is processed only once. The additional calculations for each different fluid case require only fluid matrix operations and solution processing.

User convenience was provided in the system with the implementation of several alternate solution paths and modeling options. These options, which allow a wide variety of problem types and provide the user with efficiency and accuracy trade-offs, are summarized below.

- The Direct formulation option uses structure grid point coordinates and free surface displacements as solution degrees of freedom. The structural matrices may be reduced using the NASTRAN matrix condensation technique (ASET or ØMIT data) for more economical processing of large-order problems.
- The Modal formulation option uses the mode shapes of the empty structure as generalized solution coordinates. With this option, the structure may be represented accurately by relatively small matrices.
- Gravity effects are provided which affect both the free surface displacements and the structure-fluid interface. Free body motion of the whole system is not constrained as in some other methods. The gravity effects may also be ignored by simply providing single-point constraints on the free surface pressure coordinates.
- Symmetric systems with symmetric or anti-symmetric solution cases may be modeled by providing single-point constraints on structure displacement and fluid pressure degrees of freedom.
- Compressibility effects may be provided for cases in which the fluid is completely enclosed. (Any constraints on the fluid pressure coordinates will act as an opening.) A factor is provided to define the overall pressure versus volume change. An alternate is provided to constrain the volume change to zero for pure incompressibility.
- Restart logic is provided in the DMAP (Direct Matrix Abstraction Program) to allow changes in the fluid model without reformulating the structure matrices, or generating structure modes. A user parameter provides this control, independent from the NASTRAN logic.

All of the above capabilities were specifically designed for the large-order finite element models anticipated for use in the analysis of the Space Shuttle tanks. In many cases, the actual computer hardware availability would be a significant factor in solving the real structures. Thus, the design contained the alternate paths to obtain either accurate results with long run times or less accurate results with shorter run times and faster turnaround.

The effort for the project was subdivided into five major tasks, each defined by delivery items and milestones. Brief descriptions of the tasks and the problems encountered are summarized below:

TASK I - Preliminary Specifications: The complete theoretical design, user inputs, and program flow was defined in this documentation. The report was delivered on schedule within one month of the contract start date of March 1, 1976.

TASK II - Phase 1 Program: The basic hydroelastic capabilities were installed in a Level15.9 version of NASTRAN. The program was installed at MSFC on the IBM 360/65 computer on schedule during the week of April 19. The basic capabilities included the fluid elements, free surface elements, and non-overlapping fluid-structure boundary definitions. Results were obtained for the hemispherical tank test problem.

TASK III - Phase 2 Program: The final version of the program was installed on the Level 16.0 version of NASTRAN. The delivery, on August 9, 1976, to MSFC was delayed approximately one month due to delays in the delivery to MSFC of this version of NASTRAN by the NASA distribution center.

TASK IV - Demonstration and Training: This task consisted of three parts - (1) demonstration of the system using small problems with known solutions, (2) instructing MSFC personnel in the use and maintenance of the system, and (3) assisting MSFC personnel in the modeling and execution of the large-order Space Shuttle tank problems. The first two tasks were performed at MSFC during the week of August 9, in conjunction with the program delivery. The third task continued through the remainder of the contract.

TASK V - UNIVAC 1108 Conversion and Final Report: The original schedule assumed that the 1108 version of NASTRAN Level 16 would be available to NASA by October 1976. The IBM hydroelastic program would then be converted to the 1108 at MSFC by UAI. However, the actual receipt of the 1108 Level 16 NASTRAN did not occur until January 1977, delaying the project by three months. The final report, which consists of program documentation, awaited the 1108 conversion and the execution by MSFC of the large-order Shuttle tank problems. However, preliminary versions of the user documentation were delivered to MSFC in August 1976, thereby allowing knowledgeable use of the program. This final report, contained herein, completes the outstanding contract deliverables.

The theoretical approach and the detailed analytic steps used in the formulation of the problem are described in the following theoretical approach chapter. The results of the test and demonstration effort are also summarized in this volume of the report. Volume II of this report contains the descriptions of the user-supplied data to the program and instructions for its use. Volume III contains the programmer's reference material describing the new code and modifications to NASTRAN.

2.0 THEORETICAL DEVELOPMENT

In this section the theory is developed for the general three-dimensional hydroelastic analysis in NASTRAN. Both the structure and the interacting fluid will be idealized as general, three-dimensional finite element models. Effects of free surfaces and steady-state gravity will be included. The fluids are assumed to be incompressible, irrotational, and non-viscous. Small motions of both structure and fluid relative to the static solution will be analyzed.

The basic development of the finite element equations for small motions of fluids is described in Reference 1. In Reference 2, the basic equations are cast in the form of integrals representing the time derivatives of Energy and Work using the fluid pressures as the unknown coordinates. The scalar pressures, rather than three displacements, will be used as degrees of freedom at each point in the fluid, which avoids extraneous rotational motions and directly provides for incompressibility. The disadvantage is that the structure and fluid are not automatically connected at their boundary. The pressures in the fluid must be related to the displacements of the boundaries through area factors and flow relationships.

2.1 FLUID FIELD EQUATIONS

In Reference 2 the fluid field equations are developed in the form of energy integrals using principles of variational calculus. The basic result for the compressible case is the equation:

$$\delta\left[\int_V \left(\frac{1}{2\beta}\dot{p}^2 + \frac{1}{2\rho}\nabla p \cdot \nabla p\right) dV\right] - \int_s \delta p \left(\frac{1}{\rho}\nabla p\right) \cdot d\vec{S} = 0 \qquad (1)$$

where:
- p is the pressure
- $\dot{p}$ is the time derivative of pressure
- β is the bulk modulus
- ρ is the mass density
- V is the volume
- $d\vec{S}$ is an incremental surface vector (outward)
- ∇ is the vector gradient operator
- δ is the variational operator

With the incompressible fluids, the bulk modulus is assumed infinite and the $\dot{p}^2$ term disappears. On the exterior surface the pressure gradient may be replaced with the acceleration vector using the basic momentum equation:

$$\nabla p = -\rho\ddot{\vec{u}} \tag{2}$$

Equation (1) therefore becomes:

$$\delta\int_V \frac{1}{2\rho}(\nabla p \cdot \nabla p)\, dV + \int_S \delta p\ddot{\vec{u}} \cdot d\vec{S} = 0 \tag{3}$$

or

$$\delta U + \delta W = 0 \tag{4}$$

In the finite element method of solution, a set of variables, p_i, equal to the value of p at specific points, is chosen and the volume is divided into subregions, called fluid elements, with vertices defined by the location of the variables. As with structural finite elements, a "stiffness" matrix [K] is formed from the potential energy U by the equation:

$$K^f_{ij} = \frac{\partial^2 U}{\partial p_i\, \partial p_j} \tag{5}$$

The "equivalent" potential energy for each subregion is, from Eq. (5),

$$U = \int_V \frac{1}{2\rho}\nabla p \cdot \nabla p\, dV \tag{6}$$

The pressure field for each subregion (fluid element) is dependent on the pressures p_i at its vertices.

The second term of Eq. (5) defines the "load" vector, {Q}, on the boundary, which excites the fluid. For each point j on the boundary the loading value is:

$$Q_i = \frac{\partial(\delta W)}{\partial(\delta p_i)} \tag{7}$$

Since the accelerations, $\ddot{\vec{u}}$, are defined by the structure, a transformation matrix may be developed from Eq. (7) such that:

$$\{Q\} = [B]\{\ddot{u}\} \tag{8}$$

where

$$B_{ij} = \frac{\partial}{\partial p_i \; \partial u_j} \int_S pu \; d\vec{S} \tag{9}$$

Note that the units of B are area and the units of Q are volume per time squared.

If we combine Eqs. (4), (6), (7), and (8), the resulting matrix equation relating the structure and fluid coordinates is:

$$[K^f]\{p\} + [B]\{\ddot{u}\} = 0 \tag{10}$$

On the other hand, the fluid effects the structure by applying forces over the structure surface area. The incremental work done on the structure, W_s, is:

$$\delta W_s = \int_S P(\delta u \cdot d\vec{S}) \tag{11}$$

The force on each point structure degree of freedom, F_j, is obtained from the work by the equation:

$$F_j = \frac{\partial(\delta W_s)}{\partial(\delta u_j)} \tag{12}$$

On each finite boundary area the displacements u are functions of the grid point displacements u_j. Equations (11) and (12) are evaluated resulting in the transformation matrix, A, where:

$$\{F\} = [A]\{p\} \tag{13}$$

and

$$A_{ij} = \frac{\partial}{\partial u_i \; \partial p_j} \int_S pu \; d\vec{S} \tag{14}$$

Comparing Eqs. (9) and (14) we observe that

$$[A] = [B^T] \tag{15}$$

If $[M^S]$ and $[K^S]$ are the mass and stiffness matrices of the structure, the matrix equation for the structure coordinates is

$$[M^S]\{\ddot{u}\} + [K^S]\{u\} - \{F\} = \{0\} \tag{16}$$

or, from Eq. (12):

$$[M^S]\{\ddot{u}\} + [K^S]\{u\} - [A]\{p\} = \{0\} \tag{17}$$

Equations (10) and (17) become the system of equations for a solution.

2.2 FINITE FLUID ELEMENTS

Three types of fluid elements are used to represent the three-dimensional fluid: the 4-point tetrahedron, the 6-point "wedge," and the 8-point hexahedron. The wedge and hexahedron elements are fabricated from three and ten tetrahedra, respectively, as shown in Figure 1. The pressure function within each tetrahedron is assumed to be a linear function in three directions, or:

$$p = q_o + q_1 x + q_2 y + q_3 z \tag{18}$$

The transformation between the pressures at the grid points P_i, $i = 1,2,3$ and 4, and the coefficients q may be obtained from the matrix equation:

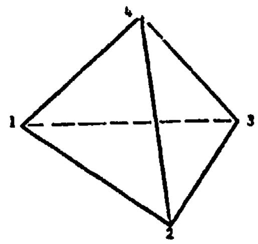
$$\{p\} = \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{bmatrix} \begin{Bmatrix} q_o \\ q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{19}$$

The matrix in Eq. (19) may be inverted, giving the matrix H, where

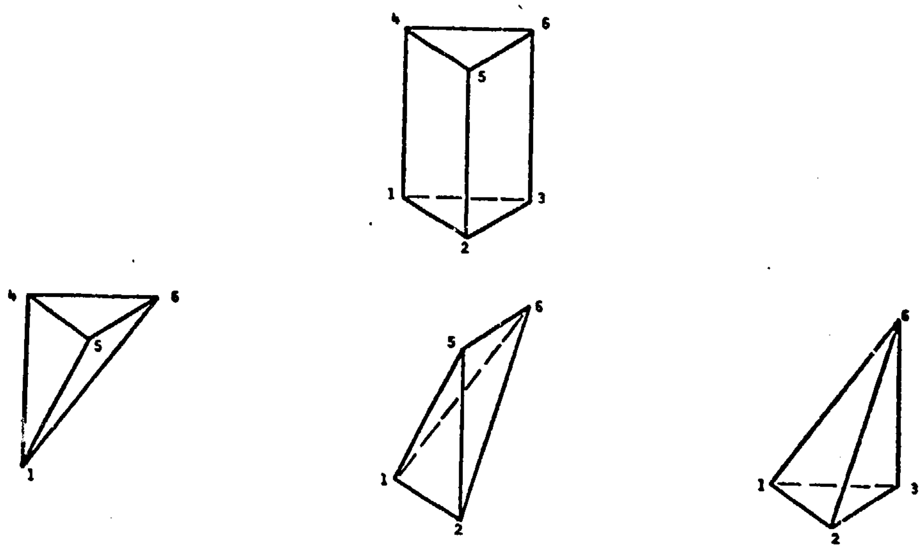
$$\{q\} = [H]\{p\} \tag{20}$$

or

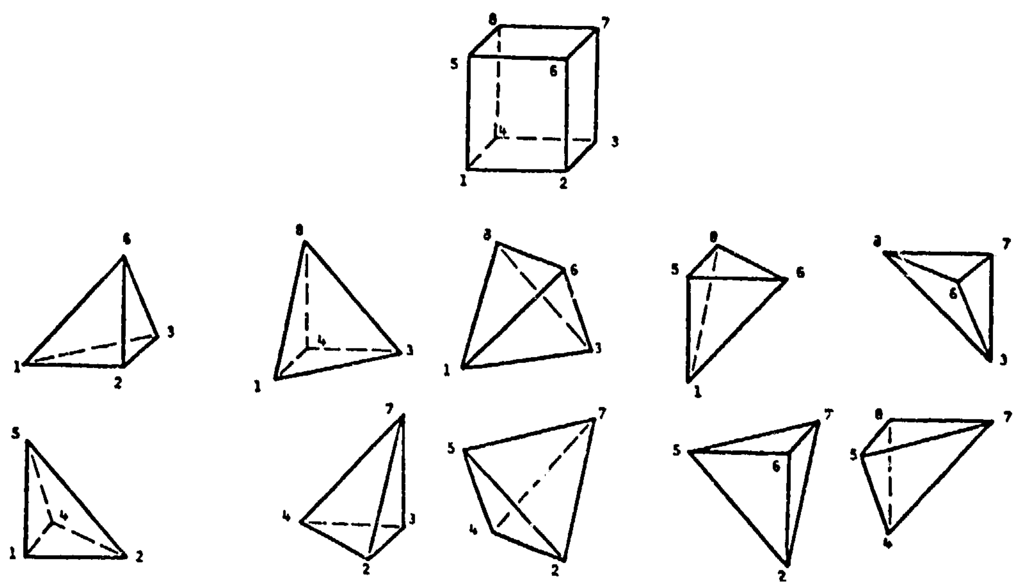
$$q_K = \sum_j H_{Kj} p_j \tag{21}$$



(a) Tetrahedron.



(b) Wedge and One of its Six Decompositions.



(c) Hexahedron and its Two Decompositions.

FIGURE 1. THREE-DIMENSIONAL FLUID ELEMENTS

The vector gradient of the pressure is obtained from Eq. (18), giving:

$$\nabla p = q_1\vec{i} + q_2\vec{j} + q_3\vec{k} \tag{22}$$

From Eq. (5) in the previous section, the energy function, U in the element is:

$$U = \frac{1}{2}\int_{Vol}\frac{1}{\rho}(\nabla p \cdot \nabla p)\, dV = \frac{1}{\rho}(q_1^2 + q_2^2 + q_3^2) \cdot Vol \tag{23}$$

From Eq. (6), the stiffness matrix terms for the element are:

$$K_{ij} = \frac{\partial^2 U}{\partial p_i\, \partial p_j} \tag{24}$$

Using the "chain" rule for differentiation, we obtain

$$K_{ij} = \frac{1}{2}\sum_{k=1}^{3}\sum_{\ell=1}^{3}\frac{\partial^2 U}{\partial q_k\, \partial q_\ell}\,\frac{\partial q_K}{\partial p_i}\,\frac{\partial q_\ell}{\partial p_j} \tag{25}$$

and from Eq. (21):

$$K_{ij} = \sum_{k=1}^{3}\sum_{\ell=1}^{3}\frac{1}{\rho}\,\delta_{k\ell}H_{ki}H_{\ell i} \tag{26}$$

(Note: $\delta_{k\ell} = 0$ if $k \neq \ell$ and $\delta_{k\ell} = 1$ if $k = \ell$)

Equation (26) may be cast as a matrix product defining the fluid "stiffness" matrix for the tetrahedron, $[K^f]$, as:

$$[K^f] = \frac{1}{\rho}[H^T][H] \tag{27}$$

The wedge and hexahedron elements are generated by adding the appropriate stiffnesses for the component tetrahedra.

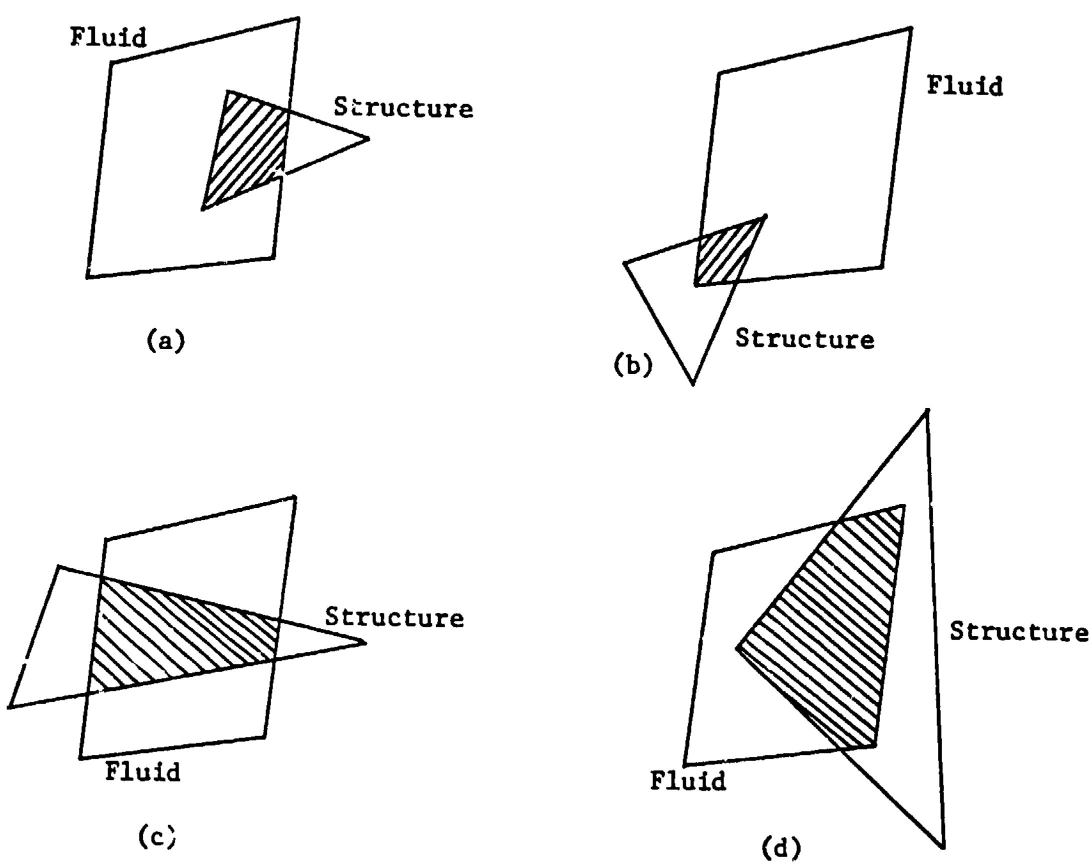
2.3 FLUID/STRUCTURE BOUNDARY MATRICES

As defined in Eq. (14) of the general development, the area matrix [A] is defined as

$$A_{ij} = \frac{\partial}{\partial u_i \ \partial p_j} \int_s \rho u \ dS \qquad (28)$$

where u and p are the displacements and pressures at the surface, S. The intersecting areas of the structure and the fluid are specified by the user as fluid-structure element pairs. From elementary geometry, the locations of the fluid points and the structure points are obtained in a coordinate system on the fluid face. Equation (28) is evaluated for each intersecting area of structure and fluid. For simplicity, only triangular structure elements are considered below. Quadrilateral elements are treated as four overlapping triangles.

Several possible examples of overlapping areas are shown in the sketch below.



Clearly, the number of combinations is too numerous to identify each case and provide a specific set of equations. Rather, a general algorithm will be developed below. The basic logical steps, listed below, will apply to all cases.

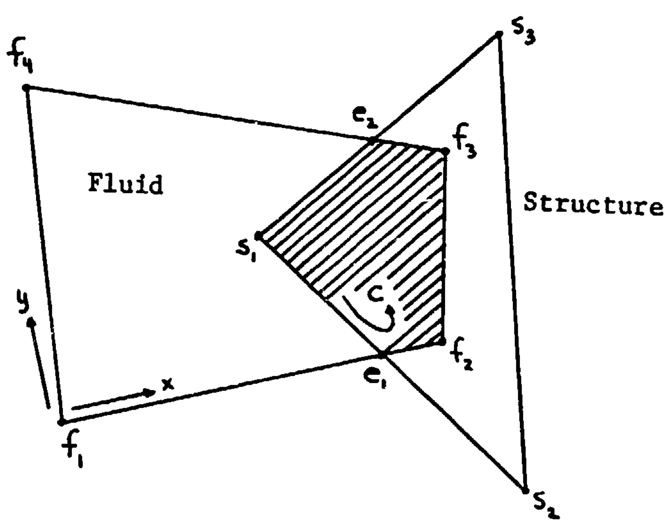
1. Test the location of each structure point and determine if it is inside the fluid area.

2. If all structure points lie outside the area, search for any intersection points, e_i, where two lines cross.

3. If no intersections occur, one of the areas encloses the other area or they are disjoint. For the first case, the inside set of points determines the boundary and the search below is skipped. When the areas are disjoint, a fatal error has occurred.

   If intersections occur, the overlapping area is determined as follows.

4. Starting with a common edge point or the location of a structure point lying in the fluid area, the points describing the polyhedron area are calculated. An example is shown in the sketch below.



The list of points, s, defining the area are:

s_1 - structure point

e_1 - edge point

f_2 - fluid point

f_3 - fluid point

e_2 - edge point

The area of the polyhedron is obtained from the line integral:

$$A = -\oint y \, dx \tag{29}$$

5. Using the finite element displacement functions, the coefficients $C_{sj}(x_s, y_s)$ are evaluated at all points on the polyhedron. The pressure applied to each point in the area is:

$$\bar{p}_s = \sum_j \tilde{C}_{sj} p_j \tag{30}$$

where p_s is the pressure on each polygon point and p_j is the pressure at corner j of the fluid element.

In triangular fluid elements the pressure distribution is

$$p = q_o + q_1 x + q_2 y \tag{31}$$

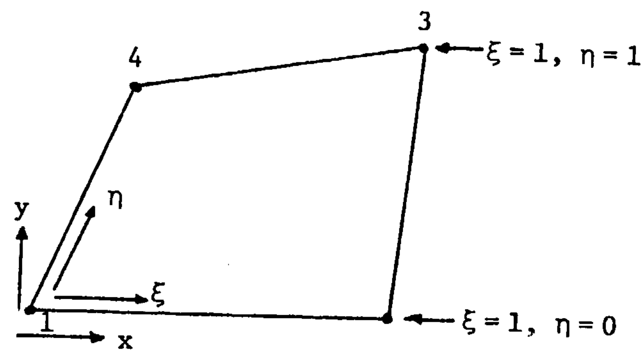
Evaluating the equation at the four corners results in the formula:

$$\begin{aligned} q_o &= p_1 \\ q_1 &= \frac{x}{x_2}(p_2 - p_1) \\ q_2 &= \left[\frac{1}{y_3} p_3 - p_1 - \frac{x_3}{x_2}(p_2 - p_1)\right] \end{aligned} \tag{32}$$

The coefficients C(x,y) are therefore:

$$\begin{aligned} C_{i1} &= 1 - \frac{x_i}{x_2} + \frac{y_i}{y_3}\left(\frac{x_2}{x_3} - 1\right) \\ C_{i2} &= \frac{x_i}{x_2} - \frac{y_i}{y_3}\left(\frac{x_2}{x_3}\right) \\ C_{i3} &= \frac{y_i}{y_3} \end{aligned} \tag{33}$$

On quadrilateral areas, an isoparametric distribution of pressure is used. The isoparametric coordinates are shown in the sketch below.



The pressure distribution is:

$$p = f_1 p_1 + f_2 p_2 + f_3 p_3 + f_4 p_4 \tag{34}$$

where:

$$\begin{aligned} f_1 &= (1 - \xi)(1 - \eta) \\ f_2 &= \xi(1 - \eta) \\ f_3 &= \xi\eta \\ f_4 &= (1 - \xi)\eta \end{aligned}$$

and the corresponding location definitions are

$$\begin{aligned} x(\xi,\eta) &= f_1 x_1 + f_2 x_2 + f_3 x_3 + f_4 x_4 \\ y(\xi,\eta) &= f_1 y_1 + f_2 y_2 + f_3 y_3 + f_4 y_4 \end{aligned} \tag{35}$$

If x_i and y_i for a point on the element are given, the two equations are used to obtain the two unknowns ξ and η. Eliminating η, a quadratic equation results for ξ:

$$a\xi_i^2 + b\xi_i + c = 0 \tag{36}$$

where:

$$\begin{aligned} a &= - x_2(y_3 - y_4) \\ b &= x_i(y_3 - y_4) - w_2 y_4 - y_i(x_3 - x_2 - x_4) \\ c &= x_i y_4 - y_i x_4 \end{aligned} \tag{37}$$

After ξ_i is obtained from the equation above, the value for η calculated from the equation:

$$\eta_i = \frac{y}{y_4 + \xi(y_3 - y_4)} \tag{38}$$

The resulting equation for the pressure coefficient at point i due to pressure at corner grid point j is:

$$C_{ij} = f_j(\xi_i, \eta_i) \tag{39}$$

6. The loads will be distributed to the structure points according to the location of each polyhedgon point on the structure area. The total force and center of force will be preserved. For each point, i, on the polyhedron, the load factors for the structure points at locations (x_1,y_1), (x_2,y_2), and (x_3,y_3) are obtained from the determinants of matrices as shown below:

$$f_{1i} = \frac{1}{\Delta}\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \tag{40}$$

$$f_{2i} = \frac{1}{\Delta}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_i & y_i \\ 1 & x_3 & y_3 \end{vmatrix} \tag{41}$$

$$f_{3i} = \frac{1}{\Delta}\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_i & y_i \end{vmatrix} \tag{42}$$

7. The pressures on the polygon points are integrated over the area according to the following rules:

   a. The average pressure of all points acts over one-half the area, located at the center of the area.

   b. The pressure at each point acts locally over an area of A/2N.

c. The total force at the center is divided equally among the polygon points. The local forces are applied at the adjacent points.

The resulting equation for the effective polygon area coefficients is:

$$F = \sum_s \tilde{A}_{is} p_s \tag{43}$$

where:

$$\tilde{A}_{ks} = \begin{cases} \dfrac{A}{2N(N-1)} & s \neq k \\[2ex] \dfrac{A}{2N} & s = k \end{cases}$$

The resulting area factor matrix is defined by the matrix product:

$$[A_{ij}] = [f]^T[\tilde{A}][C] \tag{44}$$

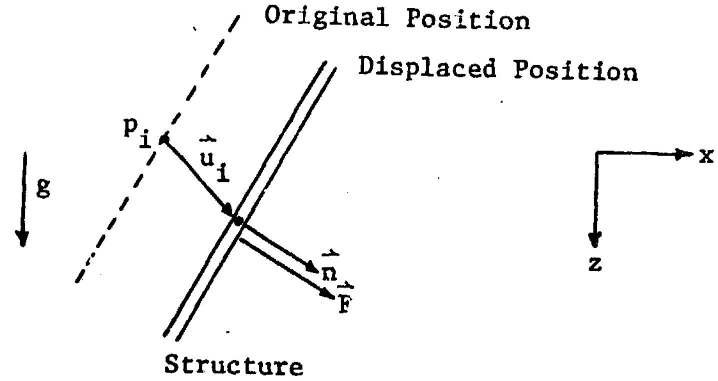
In order to provide force vectors in three dimensions, each row of the matrix A is expanded to three rows by multiplication with the unit normal vector $\vec{k}_1$.

2.4 GRAVITY EFFECTS

When a steady-state acceleration such as gravity is present in a hydroelastic problem, additional terms must be added to the fundamental equations to account for the steady-state pressure gradient. In the fluid formulation, the Euler equations assume that the pressure is defined at points fixed in space, and the fluid particles flow across the point. In the structure formulation, a Lagrange assumption is used whereby the grid points remain attached to the moving system, and the forces are applied at the displaced location. These contradicting assumptions require formulation of additional matrix terms as developed below.

2.4.1 Gravity Effects on the Structure

A change in force on the structure is illustrated in the sketch below.



The normal force, F_n, required to support the pressure is:

$$\vec{F} = -A[p_i + \rho(\vec{g} \cdot \vec{u}_i)]\vec{n} \tag{45}$$

The term Ap_i is included in the area matrices discussed previously. The second term on the right-hand side of Eq. (45) takes the form of a stiffness. The matrix takes the form:

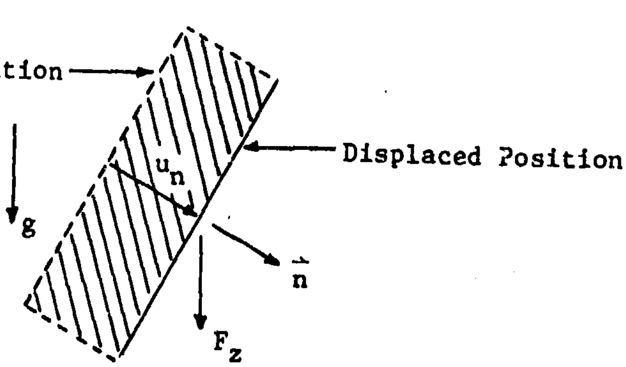
$$\begin{Bmatrix} F_x \\ F_z \end{Bmatrix} = [K]\begin{Bmatrix} u_x \\ u_z \end{Bmatrix} \tag{46}$$

where

$$[K] = -\rho|\vec{g}|A\begin{bmatrix} 0 & n_x \\ 0 & n_z \end{bmatrix} \tag{47}$$

Note that the matrix is not symmetric if $n_x \neq 0$. This violates the fundamental rule that symmetric system matrices must occur for the conservation of energy.

Another method of viewing the problem resolves the non-symmetric issue. If the structure moves, the total fluid weight changes as illustrated below.



The additional weight on the structure, w, due to the motion is:

$$w = \rho g A u_n = \rho g A(\vec{n} \cdot \vec{u}) \tag{48}$$

Since each point may move independently of the others, the increased vertical force must be applied locally and the force required to support the load is:

$$F_z = - \rho g A(\vec{n} \cdot \vec{u}) \tag{49}$$

The corresponding stiffness matrix is:

$$[K] = - \rho g A \begin{bmatrix} 0 & 0 \\ n_x & n_z \end{bmatrix} \tag{50}$$

Comparing Eqs. (47) and (50), we observe that the lower right-hand terms are equal, but the off-diagonal terms are reversed. The conclusion is that each approach missed an off-diagonal term, and the true result is:

$$[\Delta K] = - \rho g A \begin{bmatrix} 0 & n_x \\ n_x & n_z \end{bmatrix} \tag{51}$$

These stiffness terms may be processed along with the fluid-structure area coefficients described in Section 2.3. The intersecting structure fluid areas are used to define the factor A. The displacements and resulting forces are assumed to be variable on the surface, dependent on the connected grid points, and the actual stiffness will be:

$$K_{ij} = - \rho g \int_A \left[\left(\frac{\partial u_x}{\partial u_i} \frac{\partial u_z}{\partial u_j} + \frac{\partial u_z}{\partial u_i} \frac{\partial u_x}{\partial u_j} \right) n_x + \frac{\partial^2 u_z}{\partial u_i \partial u_j} n_z \right] dA \tag{52}$$

where u_x and u_z are linear functions of the grid point displacements. These integrals are evaluated in a manner similar to those developed in the area matrix calculations.

2.4.2 Free Surface Effects

A free surface is defined as a moving boundary with no restraints. When gravity effects are neglected, the boundary condition, $p = 0$, may be enforced by simply applying single-point constraints (SPC) to the input which causes the rows and columns corresponding to zero pressure to be removed from the matrix equations. However, when gravity is present we must remember that the pressure may not be zero since it is actually measured at a point fixed in space. For an upward displacement, u_f, of the free surface, the pressure at a point defined at the surface is:

$$p = \rho g\, u_f \tag{53}$$

(For a downward displacement, it is also convenient to use the same equation, measuring a fictitious negative pressure above the surface.)

In the actual solution of the free surface points, it is convenient to implement Eq. (53) in the following form:

$$-Ap + \rho g A\, u_f = 0 \tag{54}$$

where A is the free surface area associated with the fluid point. The terms in the above equation may be implemented directly into the matrix formulation. In effect, the free surface points are treated as though they were structure points, although no structural stiffness is present. The area factors A are identical to the fluid/structure interface matrices defined previously in Section 2.3. The terms ($\rho g A$) are, in effect, positive springs providing the stiffness terms, $[K_f{}^g]$, for the normal displacements, u_f, and causing the "sloshing" modes.

Furthermore, the effects of the displacements at the free surface excite the fluid in the same manner as the structure displacements. The generalized forces on the fluid are:

$$\{Q_f\} = -[A_f^g]^T\{\ddot{u}_f\} \tag{55}$$

where $[A_f{}^g]$ is a diagonal matrix of area factors connecting each free surface displacement to the corresponding pressure degree of freedom.

2.5 SYSTEM MATRIX SOLUTION

The previous development has provided the basic matrix equations to define the fluid, the fluid structure interface, and the free surface. In review, these equations are:

FLUID:

$$[K^f]\{p\} + [A_s]^T\{\ddot{u}_s\} + [A_f]^T\{\ddot{u}_f\} = \{0\} \tag{56}$$

STRUCTURE:

$$[M_s]\{\ddot{u}_s\} + [K_s + \Delta K_g]\{u_s\} - [A_s]\{p\} = \{F\} \tag{57}$$

where $[M^s]$, $[K^g]$, and $\{F\}$ are the conventional mass, stiffness, and load matrices for the structure.

FREE SURFACE

$$[K_f^g]\{u_f\} - [A_f]\{p\} = \{0\} \tag{58}$$

2.5.1 General Formulation

For the general case, when gravity is present, all the above matrices occur. The desired form of the solution matrices are:

$$[\bar{M}]\{\ddot{\bar{u}}\} + [\bar{K}]\{\bar{u}\} = \{P\} \tag{59}$$

where $\{\bar{u}\}$ is a vector containing both structure and free surface displacements and $\{P\}$ is the applied load vector. From Eq. (56), it is apparent that:

$$\{p\} = -[K^f]^{-1}[\bar{A}]^T\{\ddot{\bar{u}}\} \tag{60}$$

where $$[\bar{A}]^T = [A_s^T \mid A_f^T] \tag{61}$$

and $$\{\bar{u}\} = \begin{Bmatrix} u_s \\ \hline u_f \end{Bmatrix} \tag{62}$$

Substituting into Eqs. (57), (58), and (59), we obtain:

$$[\bar{M}] = \left[\begin{array}{c|c} M_s & 0 \\ \hline 0 & 0 \end{array}\right] + [\bar{A}][K^f]^{-1}[\bar{A}]^T \tag{63}$$

$$[\bar{K}] = \left[\begin{array}{c|c} K_s + \Delta K_g & \\ \hline & K_f^g \end{array}\right] \tag{64}$$

We observe that the matrices $[\bar{M}]$ and $[\bar{K}]$ are symmetric, and may be processed as normal structure matrices.

Unfortunately, the effect of the fluid mass terms in Eq. (63) is to fill the mass matrix, resulting in potentially time-consuming solutions for large structures. However, it is typical for large structures that a reduction procedure is employed. Defined symbolically, this reduction may be defined as:

$$\{u_s\} = [G]\{u_a\} \tag{65}$$

where the vector $\{u_a\}$ is defined by a much smaller number of degrees of freedom than $\{u_s\}$. Components of the vector $\{u_s\}$ are removed by application of constraints through the "Guyan" reduction procedure or through a modal formulation where the columns of [G] are eigenvectors of the empty structure normal modes. The structure matrices are reduced accordingly with the equations:

$$[M^a] = [G]^T[M_s][G] \tag{66}$$

$$[K^a] = [G]^T[K_s][G] \tag{67}$$

Equation (60) may be rewritten as:

$$\{P\} = [K^f]^{-1}[A]^T\{u\} \tag{68}$$

where

$$[A]^T = [G^T A_s^T \mid A_f^T] \tag{69}$$

and

$$\{u\} = \left\{\begin{array}{c} u_a \\ \hline u_f \end{array}\right\} \tag{70}$$

The reduced mass and stiffness matrices are:

$$\bar{M} = \begin{bmatrix} M^a & 0 \\ 0 & 0 \end{bmatrix} + [A][K_f^{-1}][A^T] \tag{71}$$

$$\bar{K} = \begin{bmatrix} K^a + \overline{\Delta K}^g & \\ & K_f^g \end{bmatrix} \tag{72}$$

where $$\overline{\Delta K}^g = G^T \Delta K_g G \tag{73}$$

Note that, as the size of the matrix [A] is reduced, the evaluation of the matrices for Eqs. (71) and (72) will be more economical. In the actual formulation, the columns of the matrix [A] may be treated as load vectors on the structure, and the NASTRAN reduction procedure for the load vectors may be applied directly. The gravity "stiffness" matrix $[\overline{\Delta K}^t]$ may be reduced in the NASTRAN system with the same algorithm as the mass matrix reduction process.

2.5.2 Non-Gravity Case

When the effects of gravity are ignored, the free surface is constrained such that p = 0. In this case, the free surface points are removed from the solution vector, and the solution matrices are:

$$[\bar{M}] = [M_s] + [A][\bar{K}_f]^{-1}[A]^T \tag{74}$$

$$[\bar{K}] = [K_s] \tag{75}$$

where $$\{u\} = \{u_s\} \tag{76}$$

$$[A]^T = [G]^T[A_s]^T \tag{77}$$

and $[\bar{K}_f]$ is the matrix $[K_f]$ with free surface rows and columns removed.

2.5.3 Completely Enclosed Fluid

When the fluid boundary is completely enclosed by the structure and free surfaces and no constraints are applied to the fluid points, the incompressible fluid effects must be considered. The incompressible fluid, in effect, provides a constraint on the motions of the boundary such that the net flow into the fluid is zero.

Furthermore, the fluid matrix $[K_f]$ is singular because a constant pressure defines zero flow. Mathematically, a unit pressure vector, defined as $\{I\}$, produces the result:

$$[K_f]\{I\} = \{0\} \tag{78}$$

Since the matrix $[K_f]$ has a singularity of order one, a constraint must be supplied. Because of incompressibility, we know that the total flow must be zero. The basic pressure-flow relationship is:

$$[K]\{p\} = \{Q\} \tag{79}$$

The "average" input flow is:

$$\bar{Q} = \frac{1}{N}\sum_{i=1}^{N} Q_i = \frac{1}{N} \lfloor I \rfloor \{Q\} \tag{80}$$

where $\lfloor I \rfloor$ is a row vector containing unit values.

Subtracting the average flow from each point, we obtain a flow vector, $\{Q'\}$, with a zero total value, where:

$$\{Q'\} = \{Q\} - \bar{Q}\{I\} \tag{81}$$

or

$$\{Q'\} = [[I] - \frac{1}{N}\{I\}\lfloor I \rfloor]\{Q\} \tag{82}$$

$$= [H]\{Q\} \tag{83}$$

The pressure is obtained by removing one row and column, and solving the basic matrix equation in partitioned form:

$$\left[\begin{array}{c|c} K_{11} & K_{1k} \\ \hline K_{j1} & K_{jj} \end{array}\right] \left[\left\{ \begin{array}{c} 0 \\ \hline p'_j \end{array} \right\} + \alpha \, \{I\} \right] = \left\{ \begin{array}{c} Q'_1 \\ \hline Q'_j \end{array} \right\} \tag{84}$$

where α is an undetermined constant. Noting that the {I} vector may be ignored, we obtain:

$$p'_j = K^{-1}_{jj} Q'_j \tag{85}$$

The coefficient α may be obtained by restricting the pressure result to have an average of zero, or:

$$\bar{p} = \frac{1}{N} [\lfloor I \rfloor \{p'_j\} + N\alpha] = 0 \tag{86}$$

Therefore, α is obtained from the equation:

$$\alpha = -\frac{1}{N} \lfloor I \rfloor \{p'_j\} \tag{87}$$

Note that Eq. (84) is completely satisfied if Eq. (87) is substituted into Eq. (84) and if:

$$K_{i1} + \sum_{j=2}^{N} K_{ij} = 0 \tag{88}$$

and

$$Q'_1 = \sum_{j=2}^{N} Q'_j = 0 \tag{89}$$

The first condition is satisfied by Eq. (78). The second condition is satisfied by Eq. (82), thereby producing a unique solution to the basic equation.

The resultant solution pressure vector {p} is:

$$\{p\} = \left[\begin{array}{c} -\frac{1}{N} \lfloor I \rfloor \\ \hdashline [I] - \frac{1}{N} \{I\}\lfloor I \rfloor \end{array}\right] \{p'_j\} \tag{90}$$

Observe that the pressure transformation matrix in Eq. (90) is identical to the transpose of the flow transformation, $[H_j]$, where:

$$\{Q'_j\} = [H_j]\{Q\} \tag{91}$$

$$\{p\} = [H_j]^T \{p'_j\} \tag{92}$$

and $$[H_j] = \left[-\frac{1}{N}\{I\} \,\vdots\, [I] - \frac{1}{N}\{I\}\lfloor I \rfloor \right] \tag{93}$$

The matrix "inverse" may be written symbolically as:

$$[K_f]^{-1} = [H_h]^T [K_{jj}]^{-1} [H_j] \tag{94}$$

Furthermore, it may be proven by examples that $[K_{jj}]$ may be obtained by partitioning <u>any</u> fluid point, p_1, from the matrix. If the matrix $[K_f]$ is singular (of order 1), the results are <u>exactly</u> the same regardless of the choice.

2.5.4 <u>Incompressible Fluid Restraint</u>

As described in Section 2.5.3 above, the net volume change due to boundary movement is eliminated from the fluid inertia matrix. However, the incompressibility of the fluid requires that the volume change due to structure and free surface displacements be restricted. This constraint could be implemented by supplying a constraint equation of the form:

$$\Delta Vol = \sum_i \sum_j A_{ji} u_j = 0 \tag{95}$$

or, in terms of the matrices:

$$\Delta Vol = \lfloor I \rfloor [A]^T \{u\} = 0 \tag{96}$$

For this approach, one of the displacements, u_j, is removed from the matrices, redistributing its associated mass and stiffness to the other degrees of freedom.

In the alternate method, we add a compressibility factor such that the net volume change will be small. If we define the factor, B, such that for the static case:

$$\{p\} = \{I\}\, B\, \Delta Vol \tag{97}$$

then the forces, $\{F\}$, on the structure are:

$$\{F\} = [A]\{p\} \tag{98}$$

or $$\{F\} = [K_c]\{u\} \tag{99}$$

where $$[K_c] = B[A]\{I\}\lfloor I \rfloor [A]^T \tag{100}$$

The matrix $[K_c]$ provides for the overall compressibility of the system when the fluid is completely enclosed. It is added to the system stiffness matrix and acts as a single spring connecting all surface displacement degrees of freedom.

The factor B may be obtained from the physical properties. The approximate value is:

$$B \approx \frac{\rho a^2}{Vol} \tag{101}$$

where a is the speed of sound in the fluid.

Extremely large values of B are to be avoided, since matrix numerical conditioning problems will result when the terms in the matrix $[K_c]$ are orders of magnitude larger than the structure terms. For most realistic fluid-structure combinations, this problem will not occur.

3.0 TEST RESULTS

An extensive test program was performed on the modified NASTRAN system during the coding effort and following the delivery of the system. The purpose of the program testing was to ensure correct code and validate the theoretical assumptions. In the first stage of check-out, problems consisted of simple one and two fluid element shapes in which the results could be hand-checked for correctness. This was followed by larger order, more realistic test and demonstration problems.

The choice of test and demonstration problems had to be limited to cases with known results from experimental tests and/or published analyses. Larger order detailed models representing the Space Shuttle External Tanks have also been analyzed by NASA using the program. Results of these tests are forthcoming from NASA. The basic test and demonstration problems analyzed by UAI are described below.

3.1 HEMISPHERICAL TANK TEST PROBLEM

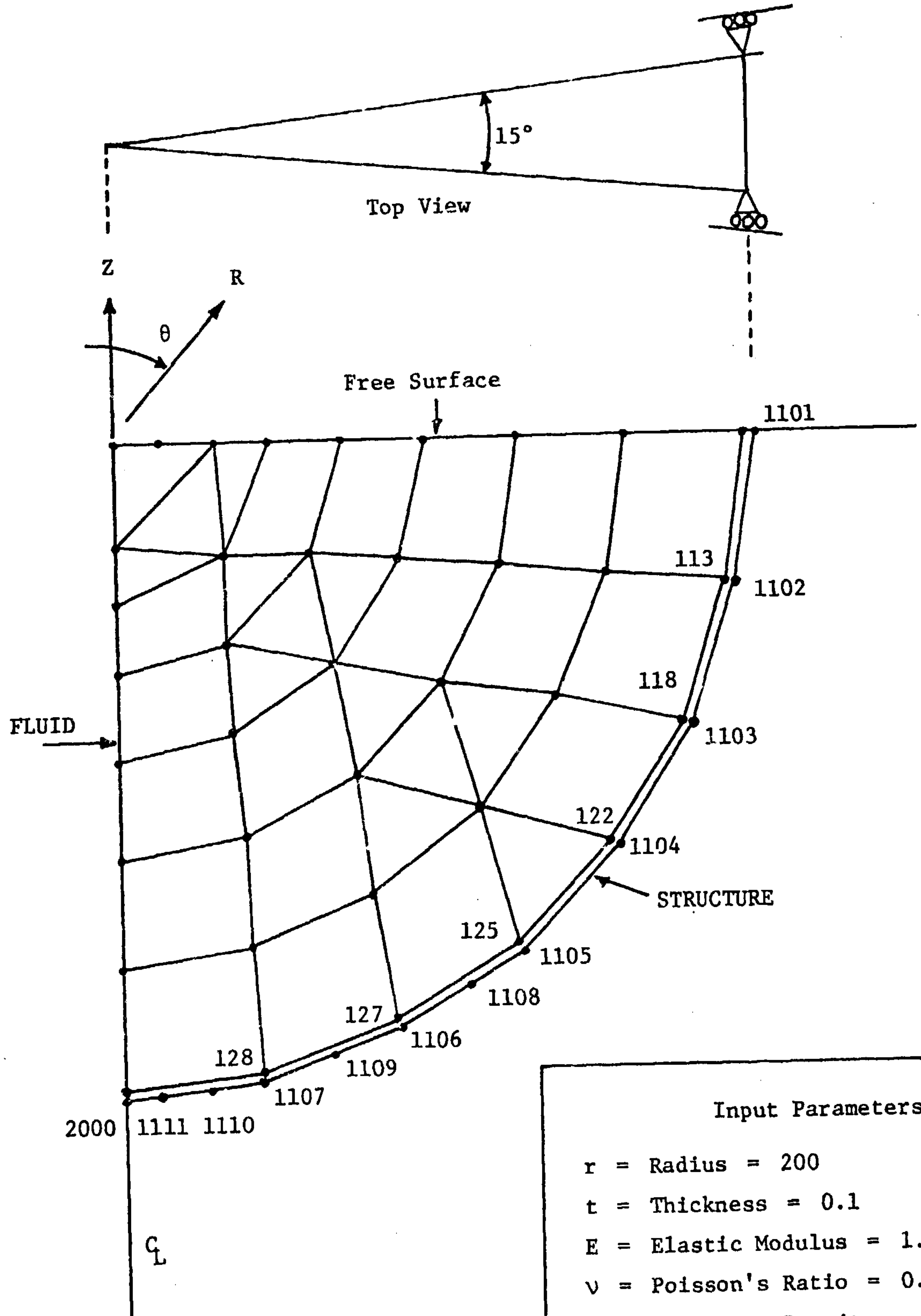
3.1.1 Problem Definition

The solution for axisymmetric sloshing and hydroelastic modes in a full hemispherical tank have been obtained by several methods of analyses [Refs. 3, 4, 5 and 6]. Although the NASTRAN program was developed for general shapes, axisymmetric geometries such as this problem may be solved by modeling a wedge-shaped section with a minimum number of elements and grid points.

The NASTRAN model, shown in Figure 2, represents a 15° sector, represented by a single layer of elements. The fluid is represented by wedge, tetrahedron, and hexahedron elements. The structure is modeled by standard NASTRAN plate elements. The large sector angle and course mesh were deliverately chosen as representative of the expected modeling practices to be used when non-axisymmetric three-dimensional problems are generated.

3.1.2 Comparison of Results

Comparisons of natural frequencies for both slosh modes and structure interactions (bulge) modes are presented for the reference studies and NASTRAN in Table 1. Slosh mode shapes are shown in Figure 3.



Input Parameters

r = Radius = 200

t = Thickness = 0.1

E = Elastic Modulus = 1.0×10^{7}

ν = Poisson's Ratio = 0.33

ρ_s = Structure Density = 2.59×10^{-4}

ρ_f = Fluid Density = 1.06×10^{-4}

g = Gravity Constant = 386.90

FIGURE 2. FIFTEEN DEGREE HEMISPHERICAL TANK MODEL

TABLE 1. COMPARISON OF NATURAL FREQUENCIES FOR VARIOUS SOLUTIONS OF HEMISPHERICAL TANK

Type	Mode	Natural Frequency - Hertz			
		Ref 3	Ref 4	Ref 5	3-D NASTRAN
Slosh	1	0.46	0.43	0.43	0.45
	2	0.62	0.62	0.62	0.67
	3	0.75	0.81	0.76	0.88
	4	0.86	1.00	0.90	1.13
Bulge	1	6.69	7.26	6.62	6.87
	2	9.92	11.47	10.25	10.99
	3	12.59	14.56	12.35	14.76

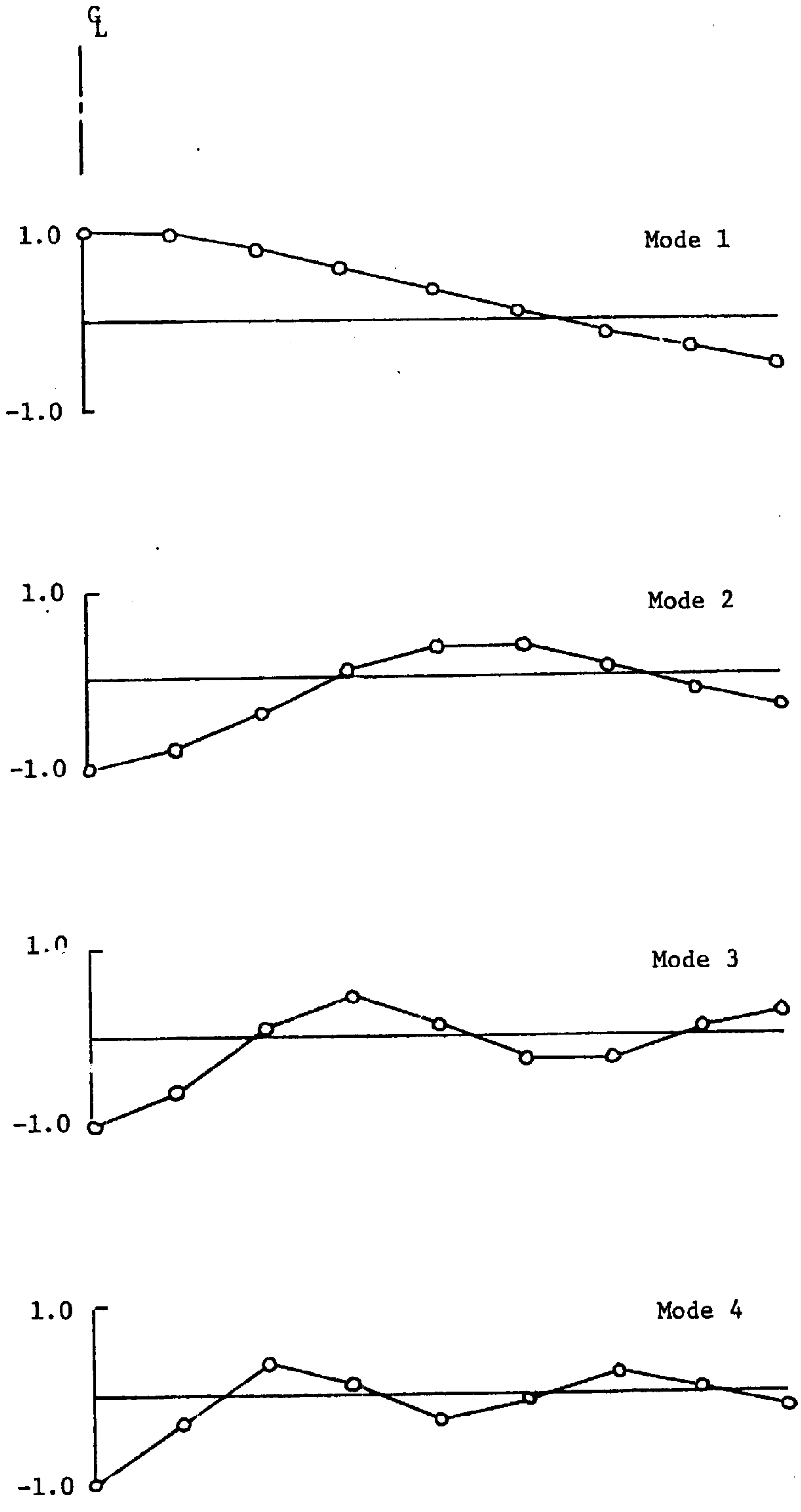


FIGURE 3. MODE SHAPES OF AXISYMMETRIC SLOSH MODES (1-4)

The modal frequencies of the present analysis compare reasonably well with those obtained by other investigators. The higher slosh mode frequencies could be expected to match more closely those presented by Guyan if the number of free surface elements was increased from 8 (NASTRAN) to 21 (Guyan). This conclusion arises from the common observation that modal frequencies tend to decrease as the model becomes more refined by increasing the number of finite elements. This explains the close match in slosh mode frequency for the first few modes and the divergence of results in the later modes. Also contributing to the differences in the NASTRAN results is the effect of representing the axisymmetric motions by modeling a 15° sector whereas the other analyses solve the axisymmetric problem directly. The actual integral over the 15° sector in NASTRAN represents a smaller area and, therefore, higher frequencies. This difference would be smaller for smaller sector angles. However, with 8 free surface elements, the present analysis results in good slosh mode frequency agreement (0-20%).

3.2 SRI TEST TANK

As a further test on the performance of the 3-D analysis of a typical problem, a series of analyses were run on a real tank model. This actual model was built and tested by Southwest Research, Inc. and the experimental results are described in Reference 6. Other analytic results were obtained using the DYNASØR axisymmetric program described in Reference 7.

The finite element NASTRAN model is shown in Figure 4. Again, a 15° sector was modeled with one layer of elements and two layers of grid points to solve for the axisymmetric modes.

The mesh size was chosen such that when it was extended to a three-dimensional half model (12 layers), the number of degrees of freedom (~ 2900) would be near the maximum for reasonable running time.

The effects of nearly all of the available options in the hydroelastic system were evaluated with the SRI model. The results are summarized in Table 2. The error ratios in terms of the test results are given in Table 3. Each of the analysis cases is described below.

<u>Test Results</u> - Obtained from Reference 6.

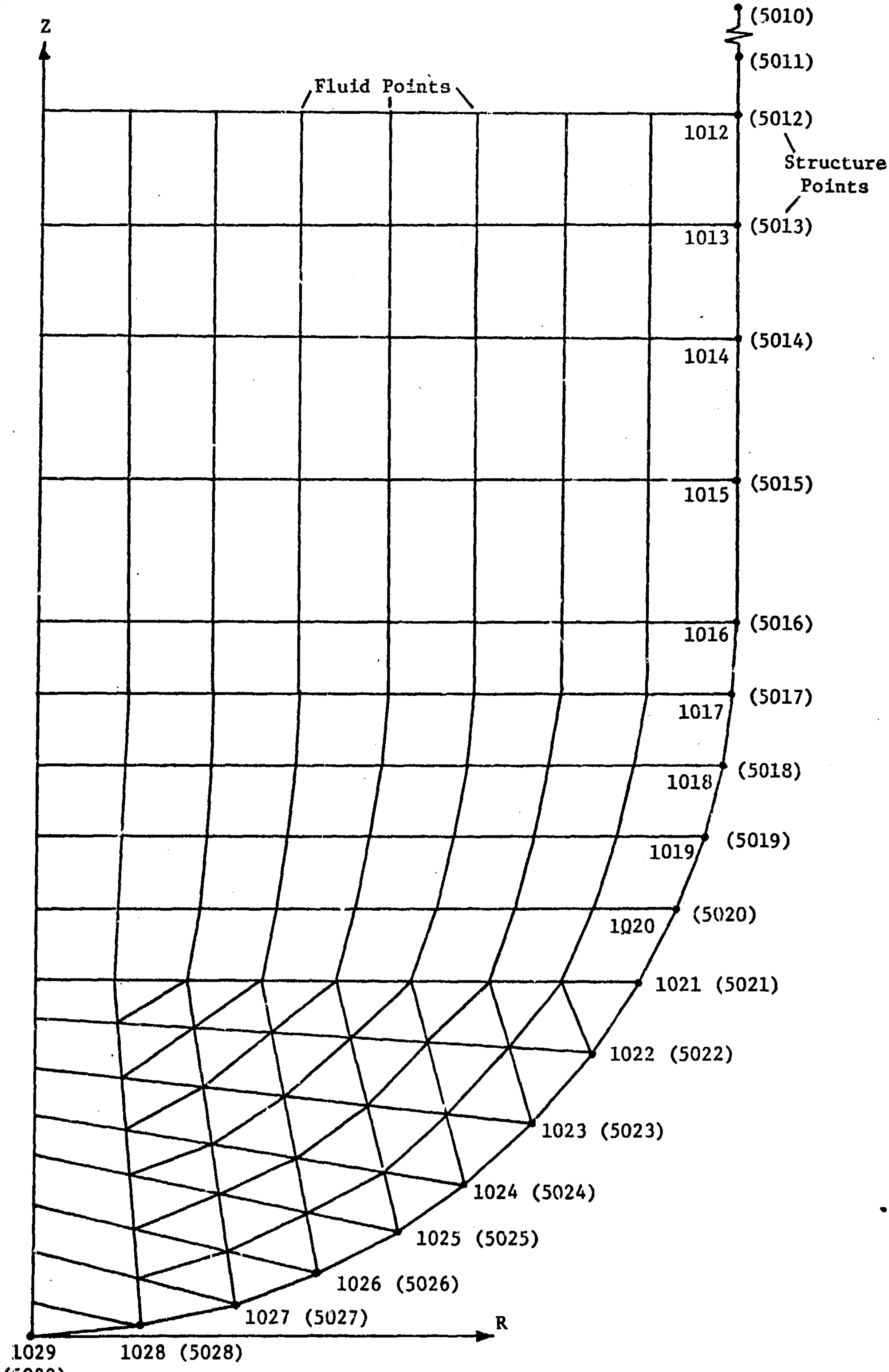


FIGURE 4. FINITE ELEMENT MODEL OF THE SRI TANK

TABLE 2. COMPARISONS OF FREQUENCIES FOR SRI TEST TANK

Analysis Case	Mode Frequencies		
	Mode 1	Mode 2	Mode 3
Test Results	495	835	1255
DYNASØR Program	531	807	1179
NASTRAN - Phase I Program			
Model A - Comp.	519	822	1239
Model B - Comp.	516	826	1239
Model B - Incomp.	541	828	1240
Model B - 1/6 Comp.	423	821	1234
NASTRAN - Phase II Program (Model B - Comp.)			
Direct - Not Reduced	513	809	1174
Direct - Reduced	612	914	1279
Direct - Ignore G	539	811	1175
Modal - 30 Modes	568	814	1185

TABLE 3. COMPARISONS OF FREQUENCY ERRORS FOR SRI TEST TANK

Run No.	Analysis Case	Frequency Difference Ratios (%)		
		Mode 1	Mode 2	Mode 3
	Test Results	0	0	0
	DYNASØR Program	7.3	-3.35	-6.1
	NASTRAN - Phase I Program			
1	Model A - Comp.	4.85	-1.56	-1.28
2	Model B - Comp.	4.25	-1.08	-1.28
3	Model B - Incomp.	9.3	-0.83	-1.20
4	Model B - 1/6 Comp.	-12.5	-1.68	-1.67
	NASTRAN - Phase II Program			
5	Direct - Not Reduced	3.7	-3.1	-6.5
6	Direct - Reduced	23.6	9.5	1.9
7	Direct - Ignore G	8.9	-2.9	-6.4
8	Modal - 30 Modes	14.8	-2.5	-5.6

DYNASØR Program - The tank was modeled and run at MSFC on the DYNASØR Program.

NASTRAN - Phase I Program - The first system delivery contained limited options and a crude method of calculating area coefficients. No overlapping structure/fluid elements were allowed. All runs were made using the direct formulation method with no matrix condensation.

1) Model A - Compressible - This model was generated by simply converting the DYNASØR data to the NASTRAN format. The mesh was similar to that shown in Figure 4 except that only four-sided elements were used. The compressibility factor was obtained from the properties of water.

2) Model B - Compressible - This was the basic test case using the model shown in Figure with overall compressibility of water. The second and third modes were excellent but the first mode was suspiciously high.

3) Model B - Incompressible - The incompressible option was used in this model to determine its effect. The first mode became worse but the second and third modes were hardly affected.

4) Model B - 1/6 Compressibility - The compressibility factor was divided by a factor of 6. The first mode frequency became lower than the test results with no change in the second and third modes. This indicated that fluid compressibility had affected the test results. The displacements in the first mode were primarily bulging of the tank, with nearly uniform vertical motion of the free surface, resulting in a net total pressure over the fluid interface. The second and third modes contained little net free surface motion and net pressure was small. The actual compressibility of the water in the test was probably lower than the theoretical factor due to aeration of the water during the vibration testing.

NASTRAN - Phase II Program - The final delivered program contained more accurate area factor calculations and the complete set of user options. The tests given below were run on this version. For comparison with the preliminary version, Model B with the calculated compressibility was used as the basic model.

5) Direct - Not Reduced - The direct method without matrix condensation was used to compare results with the Phase 1 program. Results for the first mode were improved but the second and third modes became slightly worse.

6) Direct - Reduced - In this case the solution matrices were reduced from ~257 degrees of freedom to ~60 degrees of freedom to represent only shell displacements at every other point. This reduction would be equivalent to reducing the three-dimensional model to ~300 degrees of freedom for eigenvalue extraction. All modes increased in frequency.

7) Direct - Ignore Gravity - The gravity effects were removed from the problem which reduced the solution size and the running time. The small change in results indicates that this is an efficient method for obtaining structure interaction modes. Low frequency slosh modes may not be calculated with this method.

8) Modal - 30 Modes - The modal formulation was used in this problem to reduce the structure matrices to 30 modal coordinates representing the modes of the empty structure. The error in the first hydroelastic mode was due to the fact that its shape was not well represented by the mode shapes of the empty structure. Only three of the 30 empty structure modes participated to any extent in the first mode of the combined fluid and structure systems.

3.3 TEST RESULT COMMENTS

From the experience of running the test and demonstration problems, several conclusions may be made regarding the NASTRAN hydroelastic system. These are listed below.

1. Accuracy of the system was better than expected for the mesh sizes used in the demonstration problems. With only linear elements and averaged area factors representing the fluid, three good slosh modes were obtained from only eight degrees of freedom. It appears that the accuracy for hydroelastic modes is limited more by the existing NASTRAN structure elements than by the fluid formulation. Results indicate that 15° sectors are adequate for a cylindrical or spherical shaped fluid model.

2. The results were relatively insensitive to modeling procedures. On each of the problems, different methods of subdividing the fluid space into elements were tested. For similar mesh sizes, the changes in results were insignificant.

3. The use of either Modal Formulation or Guyan reduction to condense the structural degrees of freedom tends to increase the natural frequencies of the system. For the relatively small demonstration problems, their effects on execution cost were small. However, the hydroelastic formulation produces dense solution matrices. Large order problems will require one of these reduction methods.

   The drawback to the Guyan reduction method is that it poorly represents the motions of curved surfaces with uniform loads. For best use of this method, all displacements normal to the surface should be retained. In-plane displacements may be omitted without affecting results since they are not connected to the fluid mass.

   Modal formulation is best used when the empty-structure modes are similar to the coupled fluid modes. Some of the low frequency combined-system modes do not occur as the lowest modes for the empty structure. Thus, for some cases, many modal degrees of freedom may be required to produce accurate results.

4. Although free-surface gravity effects are necessary to obtain pure sloshing modes, their effect on the hydroelastic modes is small for most problems. The alternate method of constraining the free surface pressures to zero is more efficient and requires less data input.

5. The overall compressibility factor used in the new method provides a simple, efficient manner of treating enclosed fluids. It will produce more accurate results for very stiff tanks such as those used in the SRI demonstration problem.

REFERENCES

1. R. H. McNeal (Ed.), NASTRAN Theoretical Manual, Chapter 16, NASA SP-222(01), National Aeronautics and Space Administration, Washington, D.C.

2. Zienkiewics, O. C., The Finite Element Method in Engineering Science, pp. 240-247, McGraw-Hill, New York, 1971.

3. Guyan, R. J., Ujihara, B. H. and Welch, P. W., "Hydroelastic Analysis of Axisymmetric Systems by a Finite Element Method," Proceeedings of the 2nd Converence on Matrix Methods in Structural Mechanics, Air Force Flight Dynamics Lab, AFFDL-TR-68-150, 1969.

4. Hsiung, H., "Dynamic Analysis of Hydroelastic Systems Using the Finite Element Method," PhD Dissertation, University of Southern California, Los Angeles, California, 1973.

5. Rush, R. H., "A Modal Analysis of the Fluid Shell Dynamics Interaction Problem," ME Thesis, University of Alabama, Huntsville, 1975.

6. "A Study of Axisymmetric Modes in Thin-Walled Uniform Thickness Hemispherical and Hemispherical-Spherical Tanks Containing Liquid," Final Report, Contract NAS8-30270, Southwest Research, December 1975.

7. "Dynamic Analysis of Shells of Revolution," Document No. D5-14293-1, Boeing Co. Southeast Division, Huntsville, Alabama, February 1971.